

On the production of a lepton pair in the collision of ultrarelativistic neutral particle with nonzero magnetic moment with nuclei

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Abstract

Explicit formulas which describe muon pair production in reaction $\gamma\nu \rightarrow \mu^+\mu^-\nu$ through neutrino magnetic moment are obtained and used to derive in the leading approximation cross section of muon pair production in νN -scattering due to neutrino magnetic moment. This cross section appears to be proportional to $\log^4 E_\nu$. Comparison with experimental data on tridents production provides an upper bound $\mu_{\nu_\mu} < 4 \cdot 10^{-8} \mu_B$, which is approximately two orders of magnitude weaker than that from $\nu_\mu e$ elastic scattering data.

1 Introduction

CHARM II and CCFR collaborations observed production of $\mu^+\mu^-$ -pairs in $\nu_\mu N$ -scattering data sample and soon more precise data will be available [1, 2, 3]. The data are well described by the Standard Model through the diagram shown in Fig.1, where local 4-fermion interaction represents W - and Z -exchanges. If one assumes that ν_μ has nonzero magnetic moment another mechanism of $\mu^+\mu^-$ production shown in Fig.2 comes into play. The two mechanisms of $\mu^+\mu^-$ -production do not interfere, so one should simply add probabilities, compare them with experimental data and get in this way the bound on μ_{ν_μ} .

Looking at the diagram in Fig.2 we observe that substituting an electrically charged particle instead of neutrino one obtains the reaction of the lepton pair production in the collision of two charged particles. This reaction is well studied in the literature. The first paper was published in 1934 by L.D.Landau and E.M.Lifshitz [4], who observed that the cross section of e^+e^- pair production in the collision of two nuclei grows with the energy of colliding nuclei E_N as $\ln^3 E_N$. In 60's, when a physical program for e^+e^- -colliders was prepared, the particle production by fusion diagrams was investigated in detail; for a review see [5]. In particular, subleading terms were calculated. However

as far as we know, the case when one of the colliding particles is neutral and emits photon by nonzero magnetic moment was not studied in literature. So in this paper the corresponding cross section is calculated for the first time. We observe that it increases with the energy as $\ln^4 E_\nu$, i.e. even faster than in the case of a charged particle.

The calculation of the cross section consists of two parts. In Section 2 formulas for the spectrum and for the total cross section of $\mu^+\mu^-$ pair production in the reaction $\gamma\nu \rightarrow \mu^+\mu^-\nu$ are presented in a leading logarithmic approximation. Section 3 contains the precise formulas which describe $\gamma\nu \rightarrow \mu^+\mu^-\nu$ reaction. In Section 4 with the help of the equivalent photon (Weizacker-Williams) approximation the spectrum of the muon pairs obtained in Section 2 is converted into that in reaction $\nu N \rightarrow \nu N\mu^+\mu^-$ and the total cross section of this reaction in the leading approximation is determined. In Section 5 the numerical estimates are compared with the experimental data. In Conclusions we qualitatively compare the cases of photoproduction on the charged and neutral particles and, in particular, demonstrate how $\log^4 E$ dependence in a magnetic case converts to $\log^3 E$ dependence in an electric case.

2 Reaction $\gamma\nu \rightarrow \mu^+\mu^-\nu$ in the leading logarithmic approximation

The amplitude of $\mu^+\mu^-$ -pair photoproduction on neutrino is described by two diagrams shown in Fig.3. The photon emission by neutrino is described by the following vertex:

$$V = \mu_\nu \bar{\nu} \sigma_{\alpha\beta} \nu A_\alpha q_\beta, \quad \sigma_{\alpha\beta} = \frac{1}{2}(\gamma_\alpha \gamma_\beta - \gamma_\beta \gamma_\alpha). \quad (1)$$

It is convenient to use the recursion relation for three final particles phase space:

$$d\tau_3 = \frac{1}{2\pi} d\tau_2(Q, q_1, q_2) d\tau_2(p_1 + k_1, p_2, Q) dQ^2, \quad (2)$$

where Q is the sum of the muon momenta $Q = q_1 + q_2$ (all momenta are defined in Fig.3), and to perform the integration of the square of the amplitude just in the order, shown in (2). Leaving precise formulas for the next Section, here we trace the derivation of the cross section in the leading logarithmic approximation. After the integration over the direction of μ -meson momenta which is conveniently performed in the center of mass system of two muons we come to the following double differential cross section:

$$\frac{d^2\sigma}{dQ^2 dq^2} = \frac{\alpha^2 \mu_\nu^2}{\pi} \ln\left(\frac{1+v}{1-v}\right) \frac{1}{(Q^2 + q^2)^2} \left[1 - \frac{2Q^2}{Q^2 + q^2} + \frac{2Q^4}{(q^2 + Q^2)^2} \right], \quad (3)$$

where $-q^2 = (p_1 - p_2)^2$ is virtuality of the photon and $v = \sqrt{1 - \frac{4m_\mu^2}{Q^2}}$. Integrating eq.(3) over q^2 in the domain $0 < q^2 < s - Q^2$, $s \equiv (p_1 + k_1)^2$, $Q^2 \ll s$, we obtain:

$$\frac{d\sigma}{dQ^2} = \frac{2}{3} \frac{\alpha^2 \mu_\nu^2}{\pi Q^2} \ln \left(\frac{1+v}{1-v} \right) \approx \frac{2}{3} \frac{\alpha^2 \mu_\nu^2}{\pi Q^2} \ln \left(\frac{Q^2}{m_\mu^2} \right) \quad (4)$$

Finally, integrating eq.(4) over Q^2 in the domain $4m_\mu^2 < Q^2 < s$ we get with the double logarithmic accuracy:

$$\sigma = \frac{1}{3} \frac{\alpha^2 \mu_\nu^2}{\pi} \ln^2 \left(\frac{s}{m_\mu^2} \right) . \quad (5)$$

Let us note that the log factor in eq.(4) $\ln \frac{1+v}{1-v} \approx \ln \left(\frac{Q^2}{m_\mu^2} \right)$ comes from the integration over directions of muon momenta and corresponds to $\gamma^* \gamma \rightarrow \mu^+ \mu^-$ part of the diagram (lower block in Fig.3), while the second log in eq.(5) comes from the integration over the mass of $\mu^+ \mu^-$ -pair Q^2 .

3 Exact formulas for the reaction $\gamma \nu \rightarrow \mu^+ \mu^- \nu$

In this Section precise formulas which describe the process $\gamma \nu \rightarrow \mu^+ \mu^- \nu$ shown in Fig.3 will be presented. We start from the double differential cross section $d^2\sigma/dq^2 dQ^2$, where $q^2 \equiv 2p_1 p_2$ equals the minus square of the momentum of a virtual photon and $Q^2 = (q_1 + q_2)^2$ is the invariant mass of $\mu^+ \mu^-$ -pair. Let us define the following variables: $s = (p_1 + k_1)^2$ is the Mandelstam variable; $v = \sqrt{1 - \frac{4m_\mu^2}{Q^2}}$ is μ -meson velocity in the $\mu^+ \mu^-$ pair center of mass system (for brevity instead of m_μ we will write m); finally γ is the angle between the momenta of the initial neutrino and photon in the coordinate system where the center of mass of the produced $\mu^+ \mu^-$ -pair is at rest (and where the integration of their phase space is straightforward):

$$\cos \gamma = 1 - \frac{2Q^2 s}{(Q^2 + 2p_1 p_2)(s - 2p_1 p_2)} \quad (6)$$

After all these preliminaries we can present the double differential cross section:

$$\begin{aligned} \frac{d^2\sigma}{dq^2 dQ^2} &= \frac{2\alpha^2 \mu_\nu^2}{\pi s^2 q^2} \left\{ \left\{ \frac{1}{4} \ln \left(\frac{1+v}{1-v} \right) \left\{ \frac{(s - q^2)^2}{(Q^2 + q^2)^2} [2m^2(-2 + 2\frac{m^2}{Q^2})(1 - \cos^2 \gamma) + \right. \right. \right. \\ &+ Q^2(1 + \cos \gamma) + q^2 \cos \gamma(1 - \cos \gamma) + 2\frac{m^2}{Q^2} q^2(1 - \cos^2 \gamma)] - \\ &- \left. \frac{s(s - q^2)}{(Q^2 + q^2)}(1 + \cos \gamma) + \right. \end{aligned} \quad (7)$$

$$\begin{aligned}
& + 2m^2 \frac{(s-q^2)}{(Q^2+q^2)^2} 2s(1+\cos\gamma) + \frac{2q^2}{(Q^2+q^2)^2} [2(Q^2+m^2)(s-q^2) + 2m^2q^2] \Big\} + \\
& + \frac{v}{2} \frac{(s-q^2)^2}{(Q^2+q^2)^2} \sin^2\gamma(m^2+Q^2) - \frac{Q^2vs}{2} \frac{(s-q^2)}{(Q^2+q^2)^2} (1+\cos\gamma) + \frac{1}{4}v \frac{(s-q^2)^2}{(Q^2+q^2)} \times \\
& \times [3(\cos^2\gamma-1) - 2(\cos\gamma+1)] + v \frac{s-q^2}{Q^2+q^2} [-q^2 + \frac{s}{2}(1+\cos\gamma) - \\
& - \frac{Q^2}{2} \cos^2\gamma \frac{(s-q^2)}{Q^2+q^2} + \frac{s}{2}] \Big\} \Big\} .
\end{aligned}$$

Note that the expression in the double curly brackets is proportional to q^2 for $q^2 \rightarrow 0$ (because in this limit $\cos\gamma = -1$) and cancels the pole $1/q^2$. Thus it can be presented in the form:

$$\begin{aligned}
\frac{d^2\sigma}{dq^2dQ^2} &= \frac{\alpha^2\mu_\nu^2}{\pi s^2} \left\{ \ln\left(\frac{1+v}{1-v}\right) \left\{ \frac{1}{t} (-s-2m^2) + \frac{1}{t^2} (s^2+2sQ^2+2m^2(4s+Q^2)) + \right. \right. \\
& + \frac{1}{t^3} (-2sQ^2(s+Q^2) + 2m^2(-4s^2-6sQ^2+4m^2s)) + \\
& + \frac{1}{t^4} (2s^2Q^4 + 2m^2(6s^2Q^2-4m^2s^2)) \Big\} + \\
& + v \left\{ \frac{1}{t} (s+Q^2) + \frac{1}{t^2} (-(s+Q^2)^2-6sQ^2) + \right. \\
& + \frac{1}{t^3} (8sQ^2(s+Q^2) + 4m^2sQ^2) + \frac{1}{t^4} (-8s^2Q^4-4m^2s^2Q^2) \Big\} \Big\} ,
\end{aligned} \tag{8}$$

where $t = Q^2 + q^2$.

Neglecting the terms proportional to m^2 and leaving only those terms which will produce $\log^2 s$ after the integration over Q^2 one can get equation (3) presented in Section 2.

Integrating (8) over q^2 in the interval $0 < q^2 < s - Q^2$, we obtain:

$$\begin{aligned}
\frac{d\sigma}{dQ^2} &= \frac{\alpha^2\mu_\nu^2}{2\pi Q^2} \left\{ \ln\left(\frac{1+v}{1-v}\right) \left[\frac{4}{3} + 2\frac{Q^2}{s} \ln\left(\frac{Q^2}{s}\right) - 2\frac{Q^4}{s^2} + \frac{2}{3}\frac{Q^6}{s^3} - \frac{16}{3}\frac{m^4}{Q^4} + \right. \right. \\
& + 4\frac{m^2}{s} + 8\frac{m^4}{sQ^2} + \frac{4m^2Q^2}{s^2} \ln\left(\frac{Q^2}{s}\right) - \frac{4m^2Q^2}{s^2} - \frac{8}{3}\frac{m^4Q^2}{s^3} \Big] + \\
& + v \left[\frac{2}{3} - 6\frac{Q^2}{s} + 6\frac{Q^4}{s^2} - \frac{2}{3}\frac{Q^6}{s^3} + 2\left(\frac{Q^2}{s} + \frac{Q^4}{s^2}\right) \ln\left(\frac{s}{Q^2}\right) - \right. \\
& - \frac{4}{3}\frac{m^2}{Q^2} \left(\frac{2s^3-3Q^2s^2+Q^6}{s^3} \right) \Big] \Big\} ,
\end{aligned} \tag{9}$$

and the leading term presented in eq. (4) is given by the first term in the first square brackets.

Finally, integrating (18) in the domain $4m^2 < Q^2 < s$ we obtain the expression for the total cross section of the reaction $\gamma\nu \rightarrow \mu^+\mu^-\nu$:

$$\begin{aligned}\sigma &= \frac{\alpha^2\mu_\nu^2}{2\pi} \left\{ \left\{ \frac{1}{2} \ln\left(\frac{1+v_m}{1-v_m}\right) \ln\left(\frac{s}{m^2}\right) + F\left(-\frac{1+v_m}{2}\right) - F\left(-\frac{1-v_m}{2}\right) \right\} \left[\frac{4}{3} + r + \frac{r^2}{4} \right] + \right. \\ &+ \left. \ln\left(\frac{1+v_m}{1-v_m}\right) \left[-\frac{19}{9} + r - \frac{r^2}{4} + \frac{7}{72}r^3 \right] + v_m \left[\frac{46}{27} + \frac{17}{27}r + \frac{7}{36}r^2 \right] \right\} = \\ &= \frac{\alpha^2\mu_\nu^2}{3\pi} \left\{ \ln^2 \frac{s}{m^2} - \frac{19}{6} \ln \frac{s}{m^2} + \frac{23}{9} - \frac{\pi^2}{3} + O\left(\frac{m^2}{s} \ln^2 \frac{s}{m^2}\right) \right\} ,\end{aligned}\tag{10}$$

where $r = 4m^2/s = 1 - v_m^2$ and

$$F(s) = \int_0^s \ln(1+x) \frac{dx}{x} .\tag{11}$$

4 Reaction $\nu N \rightarrow \mu^+\mu^-\nu N$ in the equivalent photon approximation

For the spectrum of the invariant mass of $\mu^+\mu^-$ pair produced in νN -scattering we obtain:

$$\begin{aligned}\frac{d\sigma}{dQ^2} &= \int_{Q^2/m_\mu}^{E_\nu} \left(\frac{d\sigma}{dQ^2} \right)_r \frac{2Z^2\alpha}{\pi} \ln\left(\frac{E_\nu}{\omega}\right) \frac{d\omega}{\omega} = \\ &= \frac{2}{3} \frac{Z^2\alpha^3\mu_\nu^2}{\pi^2 Q^2} \ln \frac{Q^2}{m_\mu^2} \ln^2 \left(\frac{E_\nu m_\mu}{Q^2} \right) ,\end{aligned}\tag{12}$$

where ω is the energy of the photon radiated by the nucleus, E_ν is the energy of the initial neutrino and Z is the charge of the nuclei; $(d\sigma/dQ^2)_r$ corresponds to the photoproduction by a real photon and is given by eq.(4). Virtuality of the photon emitted by a nucleus varies in the following region: $(Q^2/E_\nu)^2 < k^2 < Q^2$. Since in the leading logarithmic approximation $Q^2 \ll E_\nu m_\mu$, one can neglect a nuclear form factor for the experimentally interesting values of neutrino energies, $E_\nu \sim 20 \div 160$ GeV [1]. Integrating over $\mu^+\mu^-$ invariant mass from m_μ^2 up to $E_\nu m_\mu$ we get in the leading logarithmic approximation:

$$\sigma_{\nu N \rightarrow \mu^+\mu^-\nu N} = \frac{Z^2\alpha^3}{18\pi^2} \mu_\nu^2 \ln^4 \left(\frac{E_\nu}{m_\mu} \right) ,\tag{13}$$

the $\log^4 E_\nu$ dependence of the cross section announced in the Abstract.

Surely the same result for the total cross section can be obtained directly from eq.(5). Choosing the coordinate system where the energy of the initial neutrino is of the order of the mass of the muon, $E'_\nu \approx m_\mu$, we obtain:

$$\begin{aligned}\sigma_{\nu N \rightarrow \mu^+ \mu^- \nu N} &= \int_{m_\mu}^{E_\nu} \sigma_r(s) \frac{2Z^2\alpha}{\pi} \ln\left(\frac{E_\nu}{\omega}\right) \frac{d\omega}{\omega} = \\ &= \frac{2Z^2\alpha^3}{3\pi^2} \mu_\nu^2 \int_{m_\mu}^{E_\nu} \ln\left(\frac{E_\nu}{\omega}\right) \ln^2\left(\frac{\omega}{m_\mu}\right) \frac{d\omega}{\omega} = \frac{Z^2\alpha^3}{18\pi^2} \mu_\nu^2 \ln^4\left(\frac{E_\nu}{m_\mu}\right),\end{aligned}\tag{14}$$

which coincides with eq.(13) (here $s \approx \omega E'_\nu \approx \omega m_\mu$).

5 Comparison with experimental data and bounds on magnetic moment of muon neutrino

Production of $\mu^+ \mu^-$ -pairs in $\nu_\mu(\bar{\nu}_\mu)$ N -scattering observed by experimentalists is well described by the Standard Model (Fig.1) [1, 2]. In the CHARM II experiment [1] there were used glass target (for the target used in this experiment mean charge square per nucleus is $\langle Z^2 \rangle = 97.6^1$) and neutrino beam with mean energy 22.3 GeV. Experimental and theoretical cross sections are:

$$\sigma_{exp}^{[1]} = [3.0 \pm 0.9(stat.) \pm 0.5(syst.)] \times 10^{-41} \text{ cm}^2 \text{ per glass nucleus},\tag{15}$$

$$\sigma_{SM}^{[1]} = (1.9 \pm 0.4) \times 10^{-41} \text{ cm}^2 \text{ per glass nucleus}.\tag{16}$$

In the CCFR experiment [2] high energy neutrino beam with $\langle E_\nu \rangle = 160 \text{ GeV}$ was used. Iron ($Z=26$) was taken as a target. The number of trident events observed is

$$N(data) = 37.0 \pm 12.4,\tag{17}$$

that corresponds to the following cross section:

$$\sigma_{exp}^{[2]} = (4.7 \pm 1.6) E_\nu(\text{GeV}) \times 10^{-42} \text{ cm}^2 \text{ per Fe nucleus}.\tag{18}$$

The number of trident events predicted by the Standard Model is [2]

$$N(trident, \text{Standard Model}) = 45.3 \pm 2.3.\tag{19}$$

¹We are grateful to A.N.Rozanov for this information.

From (17)-(19) we obtain:

$$\sigma_{SM}^{[2]} = 5.75 E_\nu (GeV) \times 10^{-42} \text{ cm}^2 \text{ per Fe nucleus} . \quad (20)$$

As for error in this case, according to [2] it doesn't exceed 5% (see (19)) that is much smaller than the experimental error in (18) and we omit it.

Using these experimental data and approximate formula (14) we are able to get restriction on the magnetic moment of a neutrino. At 90% C.L. we obtain the following bounds on the muon neutrino magnetic moment:

$$\mu_\nu \lesssim 6.5 \cdot 10^{-8} \mu_B , \quad \mu_B = \frac{e}{2m_e} , \quad e = \sqrt{4\pi\alpha} , \quad (21)$$

from CHARM II experimental data, and

$$\mu_\nu \lesssim 4.0 \cdot 10^{-8} \mu_B \quad (22)$$

from CCFR experimental data, which are approximately two orders of magnitude weaker than the bound on μ_{ν_μ} obtained from $\nu_\mu e$ -scattering data [6].

As it was noted in [7] the neutrino magnetic moment can depend on q^2 in such a way, that it increases for small $q^2 \lesssim m_{\nu_\mu}^2$ and can reach at most $4 \cdot 10^{-6} \mu_B$. However, from (3) we see that the contribution of small q^2 into $d\sigma/dQ^2$ is suppressed as $q^2/Q^2 < (m_{\nu_\mu}/m_\mu)^2 < 10^{-6}$ that is why the production of muon pairs do not allow to study μ_{ν_μ} at small q^2 .

6 Conclusions

Our main result is the observation of rapid increase with the energy of the cross section of the charged lepton pair production in νN -collisions in the case of nonzero neutrino magnetic moment, $\sigma_\mu \sim \log^4 E$. Here we will show how this dependence changes to the famous $\sigma_e \sim \log^3 E$ dependence in the case of two charged particles collision [4, 5]. Unlike the magnetic moment case the vertex of the photon emission by a charged particle does not contain the photon momentum q . That is why the factor μ^2 in the spectrum described by eq.(3) converts into $1/q^2$. The logarithmic factor in (3) $\log\left(\frac{1+v}{1-v}\right) \approx \log(Q^2/m_\mu^2)$ remains, since it comes from the $\gamma\gamma^* \rightarrow \mu^+\mu^-$ blocks of the diagrams shown in Fig.3 which are universal. The integral over q^2 starts to diverge in the infrared. When γ^* is emitted by a massless neutrino, the values of q^2 can reach zero. However, for the massive electron q^2 minimum is above zero:

$$(q^2)_{min} \approx m_e^2 \frac{Q^4}{s(s - Q^2)} . \quad (23)$$

For the spectrum over Q^2 we get:

$$\left(\frac{d\sigma}{dQ^2}\right)_{\gamma e \rightarrow \mu^+ \mu^- e} \sim \ln\left(\frac{Q^2}{m_\mu^2}\right) \int_{(q^2)_{min}}^{s-Q^2} \frac{dq^2}{(Q^2 + q^2)^2 q^2} = \frac{1}{Q^4} \ln\left(\frac{Q^2}{m_\mu^2}\right) \ln\left(\frac{s^2}{m_e^2 Q^2}\right) . \quad (24)$$

Comparing with the analogous spectrum in the magnetic case given by eq. (4) we see that now one extra log persists. However integrating over Q^2 we get:

$$\sigma_{\gamma e \rightarrow \mu^+ \mu^- e} \sim \frac{1}{m_\mu^2} \ln\left(\frac{s}{m_e m_\mu}\right) , \quad (25)$$

and comparing with eq. (5) we note that one log has gone: the double log behavior changes to one log.

The last step is the conversion of the cross section of $\gamma e \rightarrow \mu^+ \mu^- e$ reaction into that of $Ne \rightarrow Ne \mu^+ \mu^-$ with the help of the equivalent photon (Weiszacker-Williams) approximation:

$$\begin{aligned} \sigma_{Ne \rightarrow Ne \mu^+ \mu^-} &= \int \sigma_{\gamma e \rightarrow \mu^+ \mu^- e}(\omega) \frac{2Z^2 \alpha}{\pi} \frac{d\omega}{\omega} \ln\left(\frac{m_\mu}{\omega \frac{m_N}{E_N}}\right) \sim \\ &\sim \frac{1}{m_\mu^2} \ln^3\left(\frac{E_N}{m_N}\right) , \end{aligned} \quad (26)$$

where $s = \omega m_e$. In this way we come to the well-known $\ln^3(E_N/m_N)$ dependence of the cross section of the lepton pair production in the collision of two charged particles [4, 5].

Finally, let us note that $\log^4 E$ behavior of the cross section of the reaction $2 \rightarrow 4$ is well known in the literature. In particular, it takes place in the $W^+ W^-$ bosons production in the reaction $e^+ e^- \rightarrow e^+ e^- W^+ W^-$ [8]. The reason for such a behavior is the absence of power decrease with energy of the $\gamma\gamma \rightarrow W^+ W^-$ cross section; this behaves like a constant unlike that of $\gamma\gamma \rightarrow \mu^+ \mu^-$ cross section which behaves like $(\log E^2)/E^2$. It is evident that the mechanism of the $\log^4 E$ behavior studied in our paper is completely different and arises from the photon emission by a neutrino magnetic moment.

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Figure Captions

Figure 1: Production of the $\mu^+\mu^-$ pair in $\nu_\mu N$ -scattering in the Standard Model.

Figure 2: Production of the $\mu^+\mu^-$ pair in $\nu_\mu N$ -scattering via neutrino magnetic moment.

Figure 3: Photoproduction of the $\mu^+\mu^-$ pair on neutrino via neutrino magnetic moment.

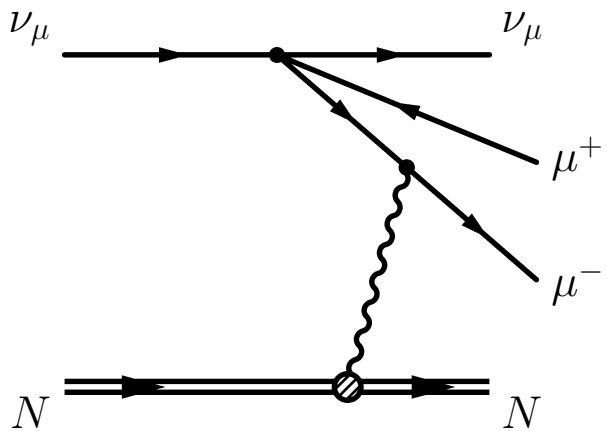


Fig.1

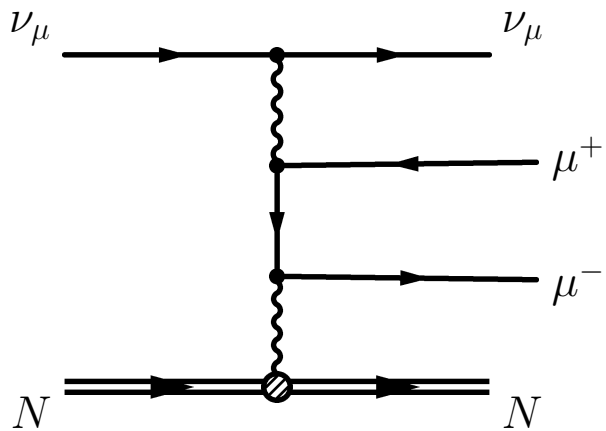
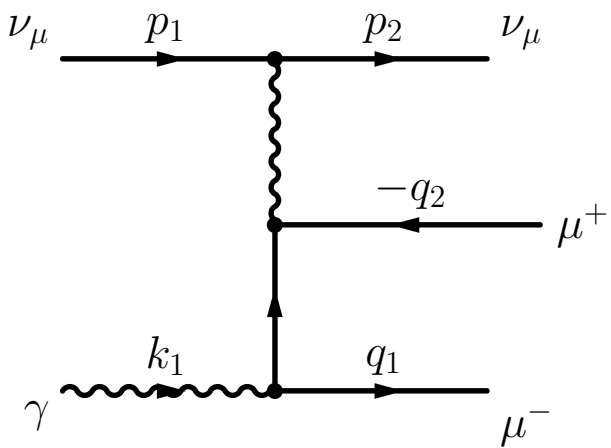
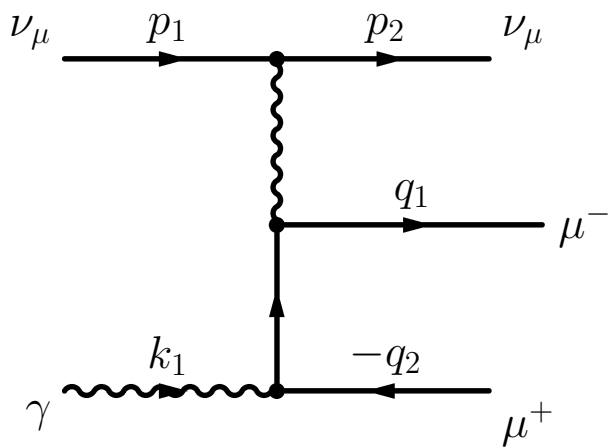


Fig.2



a)



b)

Fig. 3